**Bohr Correspondance Principle**

So in the old classical theory, we used a semi-classical quantization condition to get energy levels. We basically said that if we interpret the particle as a wave with wavelength λ, and wavefunction ψ ~ eikx-iωt, where k = 2π/λ = p/ℏ, then the only classical orbits it could exist on were ones such that an integer number of wavelengths fit within its classical path. Or another way to say it, the phase difference it accumulated along that path had to be 2πn. Could state this mathematically,



where kE(x) is the possibly position dependent wavevector of the wave. Could write this as:



This is Bohr’s quantization principle. In fact, he wrote it a little more generally. We’ll note that x and p are canonically conjugate variables, as [,] = iℏ. And so more generally postulated that the closed line integral of any two canonically conjugate variables are quantized in this fashion:



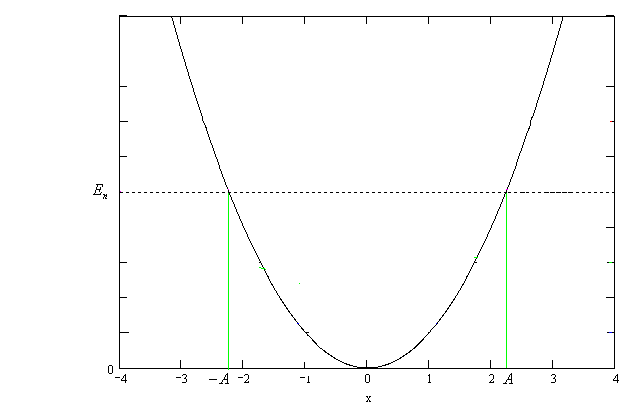
Of course it is now known that this quantization condition is not accurate. But let’s revisit a few examples nonetheless. We will see that it gives good results in the high quantum number limit, and that we can reproduce and improve its success with the forthcoming WKB approximation in the next couple files, where we learn to derive this result from the Schrodinger equation itself.

**Harmonic Oscillator**

So we have the potential:



Then suppose we’re at energy En, illustrated below:



Then our particle will oscillate back and forth between the points x = -A and x = A. These are called the classical turning points of the particle – the points where the kinetic energy of the particle drops to 0 and the particle turns back around. These occur when



And from energy conservation we know that the momentum at any point along this path is given by:



(it’s + when going right, and – when going left of course). And since x and p are canonically conjugate, we must have:



Recalling what A is, and that angular frequency of oscillation is ω = √(k/m), we can write this as:



Preferred way of writing this is:



This is almost correct of course. The actual result being:



The WKB approxiimation we’ll get in to will give us the exact result.

**Hydrogen Atom**

Now let’s do the Hydrogenic atom. We know the classical orbit is just a circle, or at least it could be. And we need to figure out the momentum of the particle in that orbit, as a function of its total energy. So classically, we must have from N2L:



So,



And now the total energy is:



So then Bohr says that around this circular orbit, we must have (generalizing to 3D, or, well, 2D since it’s a 2D orbit):



So the orbital radii are quantized. And thus the energies are:



This is in fact the correct result. The WKB approxiimation we’ll get in to will also give us the exact result for the energies. We can also derive angular momentum quantization. We have:



But we know this is not quite correct, as n is restricted to positive integers, and that would leave of the n = 0 solution we know exists. FYI, we could’ve started with an equivalent quantization relation. Since Lz = -iℏ∂/∂φ and φ are canonically conjugate too, just as px and x are, we could say:



which starts us off at the same place as before.

**Particle in Magnetic Field**

We can use this to find the energy levels of a particle in a magnetic field. Choose a (Landau) gauge, **A** = (0,Bx,0), and write H as:



where Π’s are the physical momenta. Now we take advantage of the fact that Πx and Πy are almost canonically conjugate. In fact,



So we must have:



What is the integral’s contour? It is simply a path in **Π** space with constant energy. The constant energy surface is defined by

.

The projection of this on the x-y plane is a circle with radius . And so the integral is:





since by analogy with x,y coordinates we have,



And so now we have,



and therefore, E is quantized by:



Like with the harmonic oscillator, we find that we’re off by an additional ℏωc/2. But the WKB approxiimation we’ll get in to will give us the exact result. We can also demonstrate flux quantization. y and Πy are canonically conjugate so that (yeah, this time no n+1/2, ‘cause sometimes it does that),



Now py commutes with H, as we see. Therefore it is a constant of the motion along a classical trajectory here. And so the integral pydy ought to go to 0 since py is constant and dy will just circle back onto itself. Therefore we are left with,



Now adding the Axx integral we would get 0 since Ax = 0. Therefore we may generalize to say:



And of course ∮**A**·d**r** = ∫**B**·d**S** (elementary application of Stokes’ theorem) where S a surface bounded by the contour in the first integral. And this itself is of course the magnetic flux going through the surface. And so we have:



where we (re)define the flux quantum is Φ0 = h/|e. This formula kind of matches up with what we found solving the problem exactly in the symmetric gauge. At least we found that the flux was an integer times the flux quantum. I don’t think we can interpret n here as either the n or N from the Symmetric gauge file, necessarily. In fact it would seem we need nhere = (2|n|+m)there. This consequently gives us the radius, Rc, of the orbit since, as the name suggests,



and therefore we have,

